**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: B:0.2676

Given *μ* = 45, *σ* =8

Z=X-*μ/σ*

*=0.2676*

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: A is false

Mean=38

S.D= 6

Z score =value-mean/S.D

Z score of 44=44-38/6=1

=84.13%

Above 44 age=100-84.13=15.87%

= 63 out of 400

Z score of 38=38-38/6=0

=50%

Hence people between 38&44=84.13-50=34.13%

= 137out of 400

More than 44

So false

Z score for 30= 30-38/6

=-1.33

=9.15%

=36 out of 400

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans)True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: As we know that if X ∼N(μ1, σ1^2 ), and Y ∼N(μ2, σ2^2 ) are two

independent random variables then X + Y ∼N(μ1 + μ2, σ1^2 + σ2^2 ) , and

X − Y ∼N(μ1 − μ2, σ1^2 + σ2^2)

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear

combination of X and Y ,

then Z ∼N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2 ).

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(μ + μ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4μ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: .The probability of random variable taking a value between them is 0.99.

The Probability of getting value between a and b

should be 0.99.

By finding the Standard Normal Variable Z (Z Value), we can calculate the Xvalues.

Z=(X-μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Given that P1 ~ N(5, 32 )

P2 ~N(7, 42)

Assume that $1 = Rs. 45

rupee range((centered on the mean) in 95% probability for the annual profil

of the company is

print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: the 5th percentile of profit (in Rupees) for the company

X= 540+(-1.645)\*(225)

print('5th percentile of profit (in Million Rupees) is',np.round(X,))

5th percentile of profit (in Million Rupees) is 170.0

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Probability of Division 1 making a loss P(X<0)

stats.norm.cdf (0,5,3)

0.0477903522728147

Probability of division 2 making loss(X<0)

Stats.norm.cdf(0,7,4)

0.040059156863817086